ASSIGNMENT-01
Differential Equations & Mathematical Models

Last Date of Submission: 26-04-2022, 23:59 Hours, Tuesday (in the Google classroom)

In Problems 1 through 12, verify by substitution that each given function is a solution of the given differential equation. Throughout these problems, primes denote derivatives with respect to x.

1. $y' = 3x^2;\ y = x^3 + 7$
2. $y' + 2y = 0;\ y = 3e^{-2x}$
3. $y'' + 4y = 0;\ y_1 = \cos 2x,\ y_2 = \sin 2x$
4. $y'' = 9y;\ y_1 = e^{3x},\ y_2 = e^{-3x}$
5. $y' = y + 2e^{-x};\ y = e^x - e^{-x}$
6. $y'' + 4y' + 4y = 0;\ y_1 = e^{-2x},\ y_2 = xe^{-2x}$
7. $y'' - 2y' + 2y = 0;\ y_1 = e^x \cos x,\ y_2 = e^x \sin x$
8. $y'' + y = 3\cos 2x,\ y_1 = \cos x - \cos 2x,\ y_2 = \sin x - \cos 2x$
9. $y' + 2xy^2 = 0;\ y = \dfrac{1}{1 + x^2}$
10. $x^2 y'' + xy' - y = \ln x;\ y_1 = x - \ln x,\ y_2 = \dfrac{1}{x} - \ln x$
11. $x^2 y'' + 5xy' + 4y = 0;\ y_1 = \dfrac{1}{x^2},\ y_2 = \dfrac{\ln x}{x^2}$
12. $x^2 y'' - xy' + 2y = 0;\ y_1 = x\cos(\ln x),\ y_2 = x\sin(\ln x)$

In Problems 13 through 16, substitute $y = e^{rx}$ into the given differential equation to determine all values of the constant r for which $y = e^{rx}$ is a solution of the equation.

13. $3y' = 2y$ **14.** $4y'' = y$ **15.** $y'' + y' - 2y = 0$ **16.** $3y'' + 3y' - 4y = 0$

In Problems 17 through 26, verify that $y(x)$ satisfies the given differential equation. Then determine a value of the constant C so that $y(x)$ satisfies the given initial condition.

17. $y' + y = 0;\ y(x) = Ce^{-x},\ y(0) = 2$
18. $y' = 2y;\ y(x) = Ce^{2x},\ y(0) = 3$
19. $y' = y + 1;\ y(x) = Ce^x - 1,\ y(0) = 5$
20. $y' = x - y;\ y(x) = Ce^{-x} + x - 1,\ y(0) = 10$
21. $y' + 3x^2 y = 0;\ y(x) = Ce^{-x^3},\ y(0) = 7$
22. $e^y y' = 1;\ y(x) = \ln(x + C),\ y(0) = 0$
23. $x\dfrac{dy}{dx} + 3y = 2x^5;\ y(x) = \frac{1}{4}x^5 + Cx^{-3},\ y(2) = 1$
24. $xy' - 3y = x^3;\ y(x) = x^3(C + \ln x),\ y(1) = 17$
25. $y' = 3x^2(y^2 + 1);\ y(x) = \tan(x^3 + C),\ y(0) = 1$
26. $y' + y\tan x = \cos x;\ y(x) = (x + C)\cos x,\ y(\pi) = 0$

In Problems 27 through 31, a function $y = g(x)$ is described by some geometric property of its graph. Write a differential equation of the form $dy/dx = f(x, y)$ having the function g as its solution (or as one of its solutions).

27. The slope of the graph of g at the point (x, y) is the sum of x and y.

28. The line tangent to the graph of g at the point (x, y) intersects the x-axis at the point $(x/2, 0)$.

29. Every straight line normal to the graph of g passes through the point $(0, 1)$. Can you *guess* what the graph of such a function g might look like?

30. The graph of g is normal to every curve of the form $y = x^2 + k$ (k is a constant) where they meet.

31. The line tangent to the graph of g at (x, y) passes through the point $(-y, x)$.

In Problems 32 through 36, write a differential equation that is a mathematical model of the situation described.

32. The time rate of change of a population P is proportional to the square root of P.

33. The time rate of change of the velocity v of a coasting motorboat is proportional to the square of v.

34. The acceleration dv/dt of a Lamborghini is proportional to the difference between 250 km/h and the velocity of the car.

35. In a city having a fixed population of P persons, the time rate of change of the number N of those persons who have heard a certain rumor is proportional to the number of those who have not yet heard the rumor.

36. In a city with a fixed population of P persons, the time rate of change of the number N of those persons infected with a certain contagious disease is proportional to the product of the number who have the disease and the number who do not.

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