
ASSIGNMENT-01
Relation between roots and coefficients of nth degree equations

Last Date of Submission: 02-05-2022, 23:59 Hours, Monday (in the Google classroom)

01. Solve the equation $3x^4 - 40x^3 + 130x^2 - 120x + 27 = 0$, whose roots are in geometric progression.

$$\text{Ans: } \frac{1}{3}, 1, 3, 9$$

02. Solve the equation $x^4 + 15x^3 + 70x^2 + 120x + 64 = 0$, whose roots are in geometric progression.

$$\text{Ans: } -1, -2, -4, -8$$

03. Solve the equation $81x^3 - 18x^2 - 36x + 8 = 0$, whose roots are in harmonic progression.

$$\text{Ans: } \frac{2}{9}, \frac{2}{3}, -\frac{2}{3}$$

04. If the roots of the equation $x^3 - px^2 + qx - r = 0$ be in harmonic progression, show that the mean root is $\frac{3r}{q}$.

05. The equation $x^4 - 2x^3 + 4x^2 + 6x - 21 = 0$ has two roots equal in magnitude and opposite in sign, determine all the roots.

$$\text{Ans: } \pm\sqrt{3}, 1 \pm \sqrt{-6}$$

06. The equation $3x^4 - 25x^3 + 50x^2 - 50x + 12 = 0$ has two roots whose product is 2, find all the roots.

$$\text{Ans: } 6, \frac{1}{3}, 1 \pm \sqrt{-1}$$

07. Show that all the roots of the equation $x^n + p_1x^{n-1} + p_2x^{n-2} + \dots + p_{n-1}x + p_n = 0$ can be obtained when they are in arithmetic progression.

08. Find the condition which must be satisfied by the coefficients of the equation $x^3 - px^2 + qx - r = 0$, when two of its roots α, β are connected by a relation $\alpha + \beta = 0$.

$$\text{Ans: } pq - r = 0$$

09. Find the condition that the roots of the equation $x^3 - px^2 + qx - r = 0$ be in geometric progression.

$$\text{Ans: } p^3r - q^3 = 0$$

10. Find the condition that the roots of the equation $x^3 - px^2 + qx - r = 0$ be in harmonic progression.

$$\text{Ans: } 27r^2 - 9pqr + 2q^3 = 0$$
