Last Date of Submission: 04-05-2022, 11:59PM, Wednesday (in the Google classroom)
Topic: Separable, Homogeneous, Bernoulli Equations

1. Solve the differential equation $(x-4) y^{4} d x-x^{3}\left(y^{2}-3\right) d y=0$.
2. Solve the initial value problem $x \sin y d x+\left(x^{2}+1\right) \cos y d y=0, y(1)=\frac{\pi}{2}$.
3. Solve the initial value problem $\left(y+\sqrt{x^{2}+y^{2}}\right) d x-x d y=0, \quad y(1)=0$.
4. Solve the differential equation $\frac{d y}{d x}+y=x y^{3}$.
5. Solve the differential equation $(x y+2 x+y+2) d x+\left(x^{2}+2 x\right) d y=0$.
6. Solve the differential equation $\tan \theta d r+2 r d \theta=0$.
7. Solve the differential equation $v^{3} d u+\left(u^{3}-u v^{2}\right) d v=0$.
8. Solve the initial value problem (IVP) $8 \cos ^{2} y d x+\csc ^{2} x d y=0, \quad y\left(\frac{\pi}{12}\right)=\frac{\pi}{4}$.
9. Solve the differential equation $\frac{d y}{d x}-\frac{y}{x}=-\frac{y^{2}}{x}$.
10. Solve the differential equation $d y+\left(4 y-8 y^{-3}\right) x d x=0$.
11. Solve the differential equation $\frac{d y}{d x}+y=f(x)$, where $f(x)=\left\{\begin{array}{l}2,0 \leq x<1 \\ 0, x \geq 1\end{array}, \quad y(0)=0\right.$.
12. The rate at which radioactive nuclei decay is proportional to the number of such nuclei that are present in a given sample. Half of the original numbers of radioactive nuclei have undergone disintegration in a period of 1500 years.
(a) What percentage of the original radioactive nuclei will remain after 4500 years?
(b) In how many years will only one-tenth of the original number remain?
13. The population $x$ of a certain city satisfies the logistic law

$$
\frac{d x}{d t}=\frac{1}{100} x-\frac{1}{(10)^{8}} x^{2}
$$

where time $t$ is measured in years. Given that the population of this city is 100,000 in 1980 , determine the population as a function of time for $t>1980$. In particular, answer the following questions:
(a) What will be the population in 2000 ?
(b) In what year does the 1980 population double?
(c) How large the population ultimately be?

