ASSIGNMENT-02
Symmetric Functions of the Roots

Last Date of Submission: 11-05-2022, 23:59 Hours, Wednesday (in the Google classroom)

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01. Let α, β, γ are roots of the cubic equation $x^3 + px^2 + qx + r = 0$, then find the value of the following:

(a) $\sum \alpha^2$

(b) $\sum \alpha^3$

(c) $\sum \alpha^2\beta^2$

(d) $(\alpha+\beta)(\beta+\gamma)(\gamma+\alpha)$

(e) $\frac{\beta^2+\gamma^2}{\beta\gamma} + \frac{\gamma^2+\alpha^2}{\gamma\alpha} + \frac{\alpha^2+\beta^2}{\alpha\beta}$

(f) $(\beta+\gamma-\alpha)^3 + (\gamma+\alpha-\beta)^3 + (\alpha+\beta-\gamma)^3$

02. Let $\alpha, \beta, \gamma, \delta$ are roots of the biquadratic equation $x^4 + px^3 + qx^2 + rx + s = 0$, then find the value of

(a) $\sum \alpha^2\beta\gamma$

(b) $\sum \alpha^2$

(c) $\sum \alpha^2\beta^2$

(d) $\sum \alpha^3\beta$

(e) $\sum \alpha^4$

03. Find the value, in terms of the coefficients, of the sum of the squares of the roots of the equation $x^n + p_1x^{n-1} + p_2x^{n-2} + \cdots + p_{n-1}x + p_n = 0$.

04. Find the value, in terms of the coefficients, of the sum of the reciprocals of the roots of the equation $x^n + p_1x^{n-1} + p_2x^{n-2} + \cdots + p_{n-1}x + p_n = 0$.

05. Find for the cubic equation $a_0x^3 + 3a_1x^2 + 3a_2x + a_3 = 0$, the value, in terms of the coefficients, of the following symmetric function of the roots α, β, γ :-

$$(\alpha-\beta)^2 + (\beta-\gamma)^2 + (\gamma-\alpha)^2.$$

06. Let $\alpha, \beta, \gamma, \delta$ are roots of the biquadratic equation $x^4 + px^3 + qx^2 + rx + s = 0$, then prove that

$$(\alpha^2+1)(\beta^2+1)(\gamma^2+1)(\delta^2+1) = (1-q+s)^2 + (p-r)^2$$

07. Find the numerical value of $(\alpha^2+2)(\beta^2+2)(\gamma^2+2)(\delta^2+2)$, where $\alpha, \beta, \gamma, \delta$ are roots of the equation $x^4 - 7x^3 + 8x^2 - 5x + 10 = 0$.

08. If $\alpha_1, \alpha_2, \cdots, \alpha_n$ are roots of the equation $x^n + p_1x^{n-1} + p_2x^{n-2} + \cdots + p_{n-1}x + p_n = 0$, then show that

$$(\alpha_1^2+1)(\alpha_2^2+1)(\alpha_3^2+1)\cdots(\alpha_n^2+1) = (1-p_2+p_4-\cdots)^2 + (p_1-p_3+p_5-\cdots)^2$$

09. Solve the equation $x^3 - 7x^2 + 20x - 24 = 0$, if two of its roots are of the form $a \pm a\sqrt{-1}$.

10. Solve the biquadratic equation $x^4 + 4x^3 + 8x^2 - 120x + 900 = 0$, whose roots are of the form $a \pm a\sqrt{-1}, b \pm b\sqrt{-1}$.

11. If $\alpha + \beta\sqrt{-1}$ is a root of the equation $x^3 + qx + r = 0$, then show that 2α is a root of the equation $x^3 + qx - r = 0$.

12. Find the condition that the cubic equation $x^3 + px^2 + qx + r = 0$ should have two roots α and β connected by the relation $\alpha\beta + 1 = 0$.

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