(j) Deduce Poiseuille's equation for flow of a viscous liquid through a tube. Mention the simplified assumptions present in the deduction of the formula due to which the corrections are needed.

Three capillary tubes of equal length have internal diameters 2 *mm*, 4 *mm* and 6 *mm* are connected one after another. A viscous liquid flows through the system. If the pressure difference across the whole system is 2×10^5 *Pa* then find the pressure difference across the first tube.

4+2+4=10

3 (Sem-6/CBCS) PHY HE 5

2022

PHYSICS

(Honours Elective)

Paper : PHY-HE-6056

(Classical Dynamics)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer ***any ten*** from the following : 1×10=10

(a) Mention the device in a vehicle, which uses damping to perform a useful task.

(b) If $\vec{E}$ and $\vec{B}$ represent electric and magnetic fields respectively then what physical quantity does $\frac{\vec{E}\times\vec{B}}{|\vec{B}|^2}$ represent ?

(c) Draw a diagram to indicate the location of the quantity l in the equation

$$\frac{1}{r} = \frac{1}{l}[1 + \varepsilon \cos(\theta \quad \theta_0)]$$

where the quantities carry their usual meanings.

(d) In a system there are 1000 particles and 100 constraint equations. How many generalized co-ordinates will be needed to describe the system ?

(e) A liquid is coming out of two separate capillary tubes A and B of radii in the ratio $r_A : r_B = 2:3$. What is the ratio $v_A : v_B$ of the speeds of the liquid coming out of the respective tubes ?

(f) On which side, concave or convex — of a streamline the lateral pressure of the fluid is greater ?

(g) A fluid is flowing through two different tubes A and B having Reynold's numbers $R_A = 10{,}000$ and $R_B = 40{,}000$. In which tube the fluid motion has the greater probability of having a streamline flow ?

(h) Mention an experimentally observed phenomenon which confirms time dilation.

(i) If L and H are respectively the Lagrangian and the Hamiltonian of a system then write the dimension of the quantity $= \frac{H}{L}$.

(j) What are world points in Minkowski space ?

(k) If $\frac{\partial^2 y}{\partial x^2} = \left(\frac{A}{B}\right)\frac{\partial^2 y}{\partial t^2}$ represents a progressive wave, where $A = 100$ and $B = 400$ in SI units then find the speed of the wave.

(l) An experiment is performed on the earth and the inertial mass of a body is found to be 10 *kg*. The body is taken to the Moon. What will be the gravitational mass of the body on the Moon ?

(m) Mention a phenomenon in which energy is converted into mass in accordance with $E = mc^2$.

(n) Mention a conclusion which can be drawn for a particle with total energy E and linear momentum p for which

$$E^2 - p^2c^2 < 0$$

(o) What condition the Lagrangian L of a system has to satisfy for the total energy of the system to be conserved ?

2. Answer ***any five*** of the following : 2×5=10

(a) Draw the curve $x^2 - m^2 = -1$ in the Minkowski space in the proper quadrant(s), where x stands for space co-ordinates and $m = ct$ stands for time co-ordinate.

(b) A particle of mass 2 μg and charge 2 C moving with a velocity $\vec{v} = \hat{k}10\ ms^{-1}$ enters a region in which both an electric field $\vec{E} = \hat{k}100\ Vm^{-1}$ and $\vec{B} = \hat{k}0.1\ T$ then find the kinetic energy gained by the particle when it undergoes a displacement of $\vec{l} = \hat{k}2m$.

(c) A particle is moving in a curved path under the action of a central force. Its position co-ordinates at an instant t is $P(r,\theta)$. If the semi-latus rectum of the conic in which the particle moves, is equal to r then determine the shape of the orbit.

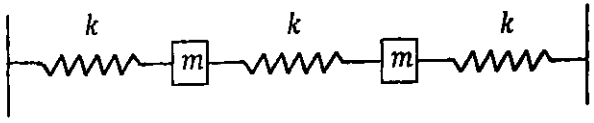
(d) The potential energy of a particle of mass 100 g capable of moving in the x direction is given by

$$V = \alpha x^2 - \beta x$$

If the particle is at rest then at what position will it be in equilibrium ? Here α and β are two positive constants.

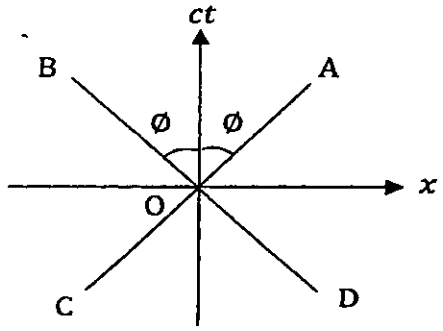
(e) Is it possible to construct a simple pendulum in the laboratory ? Justify your answer with reasons.

(f) Two identical particles, each of mass m are attached to three identical springs, each of stiffness constant k as shown in the figure. When one of the particles is slightly displaced, the system undergoes simple harmonic oscillations. What are the circular frequencies of the normal modes of vibration of the system ?



(g) A glass plate of area 100 cm^2 rests on a layer of oil of thickness 2 mm having co-efficient of viscosity 100 *poise* . Find the horizontal force needed to push the plate at a constant speed of 2 $cm\ s^{-1}$.

(h) What should be the maximum value of ϕ in the adjoining figure if a particle travelling at a speed $v < c$ and $v = c$ are represented by the world lines AC and BD ?



3. Answer the following : ***(any four)*** 5×4=20

(a) A planet of mass 2×10^{24} *kg* is moving in an elliptical orbit with major axis j and minor axis k where $j:k = 5:4$. The closest and the furthest distances of the planet from the Sun are respectively 60 million *km* and 40 million *km*. If the time period of revolution of the planet around the Sun is 3.14×10^7 *s* then find the angular momentum of the planet around the Sun.

(b) Write the definition of conservative force. Apart from gravitational force which other force is conservative in nature ? Show that for a force $\vec{F}$ to be conservative the following condition has to be satisfied :

$$\vec{\nabla} \times \vec{F} = \vec{0}$$

where $\vec{0}$ is a null vector.

(c) A particle of mass m is moving along a circle of radius R under the action of a conservative force $\vec{F} = -m\omega^2 x\hat{i} - m\omega^2 y\hat{j}$. Find the potential energy of the particle.

(d) What is called the critical velocity of a fluid in a tube ? If v_0 be the speed of a viscous fluid in streamline motion along the axis of a capillary tube of radius a and length l then find the expression for the average velocity of the fluid inside the tube.

(e) Write down the Minkowski equation of motion in terms of the four force $\vec{F}^{\mu}$ for a particle of rest mass m_0. Show that for the particle velocity $v << c$, where c is the speed of light in vacuum, the equation reduces to Newton's second law.

(f) What are time-like and space-like intervals ? If $\vec{A}^{\mu}$ is time-like then show that any other vector normal to $\vec{A}^{\mu}$ is space-like.

(g) Use the velocity four vector to derive the velocity addition theorem in special theory of relativity.

(h) A viscous fluid of density 800 *kg* m^{-3} and co-efficient of viscosity 100 *poise* is in streamline motion inside a tube of length 50 *cm* and radius R = 5 *mm*. The speed of the fluid varies with distance *r mm* from the axis of the tube as $4\left(1-\frac{r}{5\,mm}\right) mm\,s^{-1}$. Find the kinetic energy of the fluid within the tube.

4. Answer the following : ***(any four)*** 10×4=40

(a) What are generalized co-ordinates ? Mention *any two* characteristics of generalized co-ordinates. How the number of generalized co-ordinates needed for a system of particles related to the number of degrees of freedom of the system ? State the principle of virtual work and derive d'Alembert's equation from the principle of virtual work. 1+2+1+6=10

Or

Deduce Euler-Lagrange's equation from d'Alembert's principle. 10

(b) What is generalized force ? Deduce an expression for the m^{th} component of generalized force for a system consisting of n number of particles. A particle of mass m is moving in a three dimensional space under the action of a force $\vec{F} = \hat{i}F_x + \hat{j}F_y + \hat{k}F_z$ in free space. If a (x, y, z) be the co-ordinates specifying the location of the particle at an instant t then find the expression for the kinetic energy of the particle at that instant. Using the general form of Lagrange's equation arrive at the following equations

$$F_x = m\ddot{x}$$
$$F_y = m\ddot{y}$$
$$F_z = m\ddot{z}$$

1+3+3+3=10

(c) From the definition of the Hamilton of a system derive Hamilton's equations of motion. Hence, show that the Hamiltonian of the system is a constant of motion if the Lagrangian of the system is not an explicit function of time. With the help of a few mathematical steps explain the physical significance of the Hamiltonian of a system. 4+4+2=10

(d) Using Hamilton's canonical equation show that

(i) the angular momentum of a particle moving in a central force field is conserved

(ii) the particle moves according to Kepler's first law in gravitation. 5+5=10

(e) In a system N identical particles are coupled by $(N+1)$ identical springs of stiffness constant k each. If the system undergoes longitudinal oscillations then find the expression for the normal frequencies of vibration of the system. What is 'out of phase' oscillation ? Show that when the system vibrates, it generates a longitudinal wave. Find the expression for the speed of the wave. 5+1+3+1=10

(f) What is four vector ? Using velocity, momentum and force four vectors derive the Einstein mass-energy equivalence relation $E = mc^2$. 2+8=10

(g) What is Doppler effect in light ? How it is different from a similar phenomenon in sound ? What insight Doppler effect in light has given regarding the Universe ? Using four vectors find the expression for wavelength shift $\Delta\lambda$ of light emitted by a source in relative motion with the observer. A star emits light at the frequency 5×10^{17} *Hz*. If the star is receding away from the earth at a speed 3×10^3 *km* s^{-1} then find the percentage change in the frequency of the light due to Doppler effect. 1+1+1+5+2=10

(h) What is Minkowski's four dimensional space ? What is called world line in this space ? Draw the world line of a ray of light and that of moving particle in a single diagram. Show that the space-time interval is invariant under Lorentz transformation. 1+1+2+6=10

(i) A photon of energy $h\nu_0$ undergoes an elastic collision with a free electron of rest mass m_0. Using the invariance of scalar product of two four vectors under Lorentz transformation find the expression for the Compton scattering wavelength shift of the photon. 10