DEPARTMENT OF MATHEMATICS
BARNAGAR COLLEGE, SORBHOG

Home Assignment for the Paper: MAT-5404C: **Linear Algebra**

Last Date of Submission: 19-09-2026

1. Let A be an $n \times n$ matrix. If $\text{Col A} = \text{Nul A}$, show that $\text{Nul A}^2 = \mathbb{R}^n$.

2. Let V and W be vector spaces. Let $T: V \to W$ and $U: V \to W$ be linear transformations and let $\{v_1, v_2, \cdots, v_n\}$ be a basis for V. If $T(v_j) = U(v_j)$ for all $1 \le j \le n$. Show that $T(x) = U(x)$ for all $x \in V$.

3. Let $b_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, b_2 = \begin{bmatrix} -3 \\ 4 \\ 0 \end{bmatrix}, b_3 = \begin{bmatrix} 3 \\ -6 \\ 3 \end{bmatrix}$ and $x = \begin{bmatrix} -8 \\ 2 \\ 3 \end{bmatrix}$.

(a) Show that the set $\text{B} = \{b_1, b_2, b_3\}$ is a basis for $\mathbb{R}^3$.

(b) Find the change of coordinate matrix from B to the standard basis of $\mathbb{R}^3$.

(c) Write the equation that relates x in $\mathbb{R}^3$ to $[x]_{\text{B}}$.

(d) Find $[x]_{\text{B}}$.

4. The set $\text{B} = \{1+t, 1+t^2, t+t^2\}$ is a basis for the vector space P_2 of all polynomials having degree less than or equal to 2. Find $[p(t)]_{\text{B}}$, where $p(t) = 6 + 3t - t^2$.

5. Let H and K be subspaces of a vector space V. Show that $H \cap K$ is also a subspace of V and $\dim(H \cap K) \le \dim H$.

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