

DEPARTMENT OF MATHEMATICS
BARNAGAR COLLEGE, SORBHOG

Home Assignment for the Paper: MAT-5204M: **Theory of Real Functions and Riemann Integration**

Last Date of Submission: 19-09-2026

1. Let f be differentiable in $(0, \infty)$ and suppose $\lim_{x \rightarrow \infty} [f(x) + f'(x)] = L$, where L is finite. Show that $\lim_{x \rightarrow \infty} f(x) = L$ and $\lim_{x \rightarrow \infty} f'(x) = 0$.
2. For $x \in \mathbb{R}$, let $f(x) = x + \cos x \sin x$ and $g(x) = e^{\sin x} (x + \cos x \sin x)$.
 - (a) Show that $\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} g(x) = +\infty$.
 - (b) Show that $f'(x) = 2(\cos x)^2$ and $g'(x) = e^{\sin x} \cos x [2 \cos x + f(x)]$.
 - (c) Show that $\frac{f'(x)}{g'(x)} = \frac{2e^{-\sin x} \cos x}{2 \cos x + f(x)}$ if $\cos x \neq 0$ and $x > 3$.
 - (d) Show that $\lim_{x \rightarrow \infty} \frac{f'(x)}{g'(x)} = \lim_{x \rightarrow \infty} \frac{2e^{-\sin x} \cos x}{2 \cos x + f(x)} = 0$ and yet the limit $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)}$ does not exist.
 - (e) Explain why the L'Hospital's rule didn't work here?
